

University of Groningen

## **Towards a relativistic self-consistent nucleon spectral function in the nuclear medium**

Korpa, CL; Malfiet, Rudi

*Published in:*  
Physics Letters B

*DOI:*  
[10.1016/0370-2693\(93\)91602-J](https://doi.org/10.1016/0370-2693(93)91602-J)

**IMPORTANT NOTE:** You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

*Document Version*  
Publisher's PDF, also known as Version of record

*Publication date:*  
1993

[Link to publication in University of Groningen/UMCG research database](#)

### *Citation for published version (APA):*

Korpa, CL., & Malfiet, R. (1993). Towards a relativistic self-consistent nucleon spectral function in the nuclear medium. *Physics Letters B*, 315(3-4), 209-214. [https://doi.org/10.1016/0370-2693\(93\)91602-J](https://doi.org/10.1016/0370-2693(93)91602-J)

### **Copyright**

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

### **Take-down policy**

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

*Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.*

# Towards a relativistic self-consistent nucleon spectral function in the nuclear medium

C.L. Korpa and Rudi Malfliet

*Kernfysisch Versneller Instituut, NL-9747 AA Groningen, The Netherlands*

Received 27 May 1993, revised manuscript received 27 July 1993

Editor: C. Mahaux

An iterative procedure is developed for solving the Schwinger–Dyson equation for the coupled nucleon and pion propagators for the case when vertex corrections are neglected. The presence of ghost poles in the dressed propagators is avoided by using explicitly only the imaginary part of the propagators, eliminating the real part through a dispersion relation.

## 1. Introduction

The application of relativistic models to the description of nucleon scattering and nuclear matter produced encouraging results [1], however the use of perturbation theory seems not to be justified [2]. One way of going beyond perturbation theory is to find self-consistent solutions of the equations of motion, thus summing infinite classes of diagrams. To study the nucleon propagator and self-energy we thus use the Schwinger–Dyson (SD) equations, but in order to close the hierarchy we neglect vertex corrections.

We want to solve the SD system of equations self-consistently dressing both the nucleon and meson propagators. Since we are especially interested in the broadening of the antiparticle and particle peaks in this letter we consider only the effect of the pion, which is expected to play the most important role due to its strong coupling to the nucleon and its small mass.

The case with undressed meson propagators was first discussed by Wilets et al. [3] and they showed how the renormalization of the Fock diagram can be performed. For a renormalizable coupling that assures finiteness of the real part of the self-energy (the imaginary part is always finite). Since we want to use also non-renormalizable pseudovector coupling, to assure finiteness of the real part of the self-energy we

also use a form-factor for the nucleon–nucleon–pion coupling. We use the form-factor, however, only in the calculation of the imaginary part of the self-energy, calculating the real part from a dispersion relation. Even when we use a form-factor, we still work with the renormalized nucleon and pion propagators. That means that dressed propagators (in vacuum) have poles at the physical mass with the same residues as the free propagators.

An approach similar to the present one was used in ref. [4] for finding the self-consistent nucleon spectral function in vacuum, however with free meson propagators. That approach was based on the unrenormalized propagator; finiteness of the real part of the self-energy was achieved by using a form-factor in the calculation of the imaginary part. Since the equations to be solved are non-linear the use of renormalized (i.e. rescaled) propagators and unrenormalized ones produces different results.

## 2. Vacuum solution

The procedure of ref. [3] can be generalized to the case when not only the nucleon but also the mesons are dressed, leading (for renormalizable coupling) to a manifestly finite system of equations for the nucleon and meson propagators [5]. The equations involve only the imaginary parts of the propagators

since the real parts are eliminated using dispersion relations which ensure the correct analytical (and causal) behaviour of the propagator. This approach explicitly eliminates [6] the ubiquitous ghost poles from the propagators. The starting equations are those for the imaginary parts of the renormalized self-energies:

$$\text{Im } \Sigma(p)$$

$$= -2g^2 \int \frac{d^4k}{(2\pi)^4} \Gamma \text{Im } G(p+k) \Gamma \text{Im } D(k) \\ \times \Theta(p_0+k_0) \Theta(-k_0), \quad (1)$$

$$\text{Im } \Pi(p)$$

$$= -2g^2 \text{Tr} \int \frac{d^4k}{(2\pi)^4} \Gamma \text{Im } G(p+k) \Gamma \text{Im } G(k) \\ \times \Theta(p_0+k_0) \Theta(-k_0). \quad (2)$$

Here  $G$  and  $D$  are the renormalized nucleon and meson propagator and we chose  $p_0 > 0$  and used the results of ref. [7].  $\text{Im } \Sigma$  in eq. (1) denotes the imaginary parts of coefficients of Dirac gamma matrices and the same is true for the corresponding label on the right-hand side. The real parts of the self-energies are given by subtracted dispersion relations (for details see ref. [5]), the subtractions coming from mass and wave-function renormalization and imposition of renormalization conditions on the propagators. The renormalization conditions assure that the dressed propagators have a pole at the physical mass and that the corresponding residue is the same as for the undressed propagator. This determines the self-energy and its derivative at the renormalization point (i.e. physical mass). Writing  $\Sigma(p) = a(p^2)\not{p} + b(p^2)$  we then get for the real parts

$$\text{Re } a(p^2) = \frac{p^2 - M^2}{\pi} \mathcal{P} \int_{(M+\mu)^2}^{\infty} \frac{d\sigma^2 \text{Im } a(\sigma^2)}{(\sigma^2 - M^2)(\sigma^2 - p^2)} \\ - 2M^2 a' - 2Mb', \quad (3)$$

$$\text{Re } b(p^2) = \frac{p^2 - M^2}{\pi} \mathcal{P} \int_{(M+\mu)^2}^{\infty} \frac{d\sigma^2 \text{Im } b(\sigma^2)}{(\sigma^2 - M^2)(\sigma^2 - p^2)} \\ + 2M^3 a' + 2M^2 b', \quad (4)$$

$$\text{Re } \Pi(p^2)$$

$$= \frac{(p^2 - M^2)^2}{\pi} \mathcal{P} \int_{(2M)^2}^{\infty} \frac{d\sigma^2 \text{Im } \Pi(\sigma^2)}{(\sigma^2 - \mu^2)^2 (\sigma^2 - p^2)},$$

where

$$a' \equiv \frac{1}{\pi} \int_{(M+\mu)^2}^{\infty} \frac{d\sigma^2 \text{Im } a(\sigma^2)}{(\sigma^2 - M^2)^2},$$

and

$$b' \equiv \frac{1}{\pi} \int_{(M+\mu)^2}^{\infty} \frac{d\sigma^2 \text{Im } b(\sigma^2)}{(\sigma^2 - M^2)^2}.$$

$M$  and  $\mu$  are the physical mass of the nucleon and of the pion. The imaginary parts of the propagators are expressed through the free propagators and self-energies using the SD equations. The real parts of the propagators are completely eliminated from the iteration procedure, but in deriving the above equations (1) and (2) the fact that they are related to the imaginary parts through dispersion relations has been used [7].

For pseudoscalar coupling the integrals in the above expressions for the real parts of the self-energies are convergent, while for the pseudovector coupling we use a form-factor which softens the behaviour of the imaginary part of the self-energy, thus also assuring convergence of the integrals. We use a covariant form-factor (multiplying the standard point coupling) given by

$$F(k; p, p+k) = \frac{\Lambda_\pi^2 + \mu^2}{\Lambda_\pi^2 + k^2} \frac{\Lambda^4 + M^4}{\Lambda^4 + (p^2)^2} \frac{\Lambda^4 + M^4}{\Lambda^4 + [(p+k)^2]^2}, \quad (6)$$

where  $k$  is the momentum of the pion and  $p$  and  $p+k$  of the nucleons. Relatively little is known about the dependence of the form-factors on the nucleon momenta, but it was noted in refs. [4,8] that a strong suppression is needed. We were guided by the requirement of significant suppression above the energy-momentum range of 2 GeV, implying values of the cut-off parameter  $\Lambda$  around 1.2 GeV, which we use in the following for both pseudoscalar and pseudovector coupling (except for the vacuum solution shown in fig. 1a, where, for illustrative purposes, no form-factor was used).

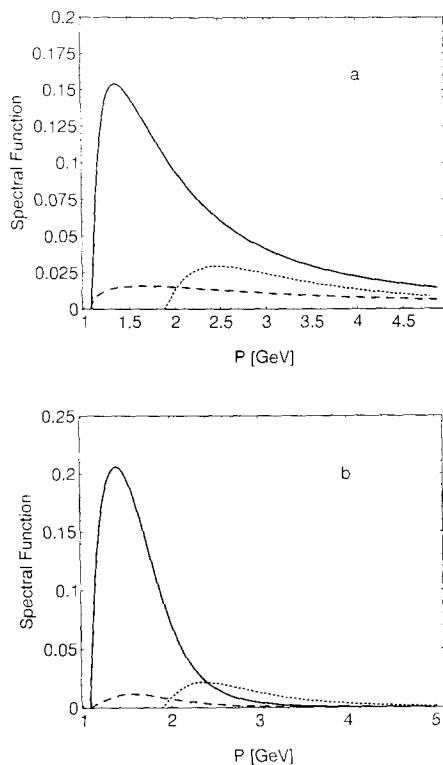


Fig. 1. The spectral functions of the nucleon and pion, (a) for pseudoscalar point coupling and (b) pseudovector coupling with  $\Lambda = 1.2$  GeV,  $\Lambda_\pi = 0.8$  GeV. The pion spectral function (dotted line) has been multiplied by 10 and the units are in  $\text{GeV}^{-2}$ . The full line shows  $Pp_1 + p_2$  and the dashed one  $Pp_1 - p_2$  (units are in  $1/\text{GeV}$ ), where  $P = \sqrt{p^2}$  and the nucleon spectral function is written as  $\rho_1(p^2)\not{p} + \rho_2(p^2)$ . The delta functions at the physical masses are not shown.

The closed system of equations obtained in this way was solved iteratively, starting from free propagators, with the standard value of the pseudoscalar  $\pi\text{NN}$  coupling  $g_{\text{ps}} = 13.4$  and pseudovector coupling  $g_{\text{pv}} = g_{\text{ps}}/2M$ . The spectral functions (apart from delta functions at the physical mass) are shown in fig. 1. Fig. 1a corresponds to the pseudoscalar point coupling and fig. 1b for the pseudovector one, the latter being obtained with cut-off parameters  $\Lambda = 1.2$  GeV and  $\Lambda_\pi = 0.8$  GeV. As one can see, in both cases the modification of the pion propagator due to the nucleon-antinucleon loop is very small (even though the coupling is very large) and thus the effect of dressing the pion on the nucleon spectral function is completely negligible. We find that for pseudoscalar

coupling less than 10% of the total strength for the pion comes from nucleon-antinucleon excitations, while for the pseudovector coupling that number is considerably smaller, since the form-factor cuts off the high-momentum tail. Thus, the result in fig. 1a reproduces the self-consistent solution obtained in ref. [3], where the undressed pion was used. Also, in these cases the first iteration for the nucleon propagator (the lowest order perturbation-theory contribution) differs very little from the self-consistent solution. However, that is not a general feature of the solution as, for example, for a scalar meson coupling to the nucleon with similar strength, the perturbation theory and self-consistent nucleon spectral function differ considerably.

### 3. Medium effects

In nuclear matter the situation is much more complicated, since Lorentz covariance is broken and the spectral functions depend on two variables, the energy and the intensity of the three-momentum (we consider a zero-spin system). Furthermore, though we want to concentrate on the effect of the pion, since it is known [1] that the scalar and vector meson are responsible for saturation, we want to include their effect too, but only at the mean-field (Hartree diagrams) level.

In vacuum the Hartree diagram (tadpole) coming from coupling to the scalar meson does not have any effect, since it contributes an (infinite) constant which is absorbed in the mass renormalization. In the case of nonzero baryon density the tadpoles (a vector meson also contributes) cannot be simply thrown away, since their effect is density dependent. As already noticed in ref. [4] in the scheme which we use they cause additional difficulties, since their imaginary part is zero. In principle one can still perform a renormalization which produces a finite scalar-tadpole contribution [9] (the vector-meson tadpole is finite without renormalization, since the vector meson couples to the conserved baryon current), but the complications are so great that one needs a more practical approach. One way out is to use a small-density expansion, which means a linear increase of the scalar-tadpole contribution with density. Alternatively, one can introduce a form-factor which would

make the tadpole finite. Choosing the value of the cut-off (operating in the Euclidean region) one could aim to adjust the binding energy and saturation density to the observed value, thus determining the scalar and vector tadpoles. In view of the ambiguity in such a procedure (we have to assume the Euclidean form-factor) we do not attempt it here, but use the value of the scalar tadpole as a parameter in the calculation. The results presented below are obtained with its value being  $-0.14$  GeV at saturation density ( $p_F = 0.27$  GeV). The value of the vector tadpole must be proportional to the baryon density, but we leave it out from our results since it only introduces a shift in the energy variable.

First, we want to establish the high-three-momentum (i.e.  $|p| \rightarrow \infty$ ) limit of the nucleon's in-medium spectral function. In the following consideration we specialize to the case when the mesons are not dressed. Then all diagrams in which the large momentum flows through the density dependent part of the nucleon propagator will be suppressed. Thus, the only surviving contributions are the ones already present in vacuum and the tadpoles. This means that the high-three-momentum limit of the in-medium nucleon propagator is given by the "shifted" vacuum propagator:

$$G_{\text{med}}(p_0, |p| \rightarrow \infty) = G_{\text{vac}}(p_0 - c_0, \mathbf{p}; M = M_{\text{nuc}} + c_s), \quad (7)$$

where  $c_0$  and  $c_s$  are the values of the vector and scalar tadpoles (the latter is negative).

It is advantageous to work with retarded quantities (propagators and self-energies) in this case and the equations for the imaginary parts of the retarded nucleon and meson self-energies have the form [5]

$$\begin{aligned} \text{Im } \Sigma^{(+)}(p) = & 2g^2 \int \frac{d^4k}{(2\pi)^4} \\ & \times \Gamma \text{Im } G^{(+)}(p+k) \Gamma \text{Im } D^{(+)}(k) \\ & \times [\Theta(p_0 + k_0 - E_f) \Theta(-k_0) \\ & - \Theta(E_f - p_0 - k_0) \Theta(k_0)], \end{aligned} \quad (8)$$

$$\begin{aligned} \text{Im } \Pi^{(+)}(p) = & 2g^2 \text{Tr} \int \frac{d^4k}{(2\pi)^4} \\ & \times \Gamma \text{Im } G^{(+)}(p+k) \Gamma \text{Im } G^{(+)}(k) \\ & \times [\Theta(p_0 + k_0 - E_f) \Theta(E_f - k_0) \\ & - \Theta(E_f - p_0 - k_0) \Theta(k_0 - E_f)], \end{aligned} \quad (9)$$

where  $G^{(+)}$  and  $D^{(+)}$  are the retarded nucleon and meson propagator. The limiting behaviour of the nucleon propagator expressed by eq. (7) is expected approximately to hold also in the case of the dressed pion, since a large momentum flowing through the (dressed) pion propagator suppresses the diagram in the same way as for the free pion.

This means that we can restrict the loop-momentum integration to a finite region in the three-momentum and energy, where the medium contribution to the Fock diagram is important. It is apparent that this contribution is finite, the vacuum renormalization eliminated all the infinities in the Fock diagram.

We first study the nucleon spectral function for negative energy, i.e. the antinucleons. The broadening of the mean-field delta function begins at the two-loop level, even with the free-pion propagator. This process can be interpreted as a nucleon dropping from the Fermi sea, by emission of two pions, into the Dirac sea. The process involves pions with a three-momentum of the order of the nucleon mass, thus we expect the dressing of the pion not to play an important role here. We start from the limiting result (7) as the zeroth iteration and calculate the next one in a finite energy as well as three-momentum region (but involving positive and negative energies). Then we continue the iterative procedure, but only for the negative energy region, keeping the result for positive energies fixed as obtained after the first iteration. The two energy regions almost decouple, except for possible shifts of peaks (coming from the real part of the self-energy) and slight changes in the imaginary part for negative energies.

After 4–5 iterations we obtain the converged result, which, furthermore, does not differ very significantly from the second iteration. The results are shown in figs. 2 and 3, for the pseudoscalar and pseudovector coupling, respectively. We note the much larger broadening obtained with pseudoscalar coupling (for the pseudovector one the width at half maximum never exceeds 80 MeV), even though the form-factors and cut-off parameters are the same. The results are somewhat dependent on the value of the cut-off, but the general features remain the same for cut-off values around 1 GeV. Studying antinucleon spectra in relativistic heavy-ion collisions or antiproton annihilation in nuclei should provide informa-

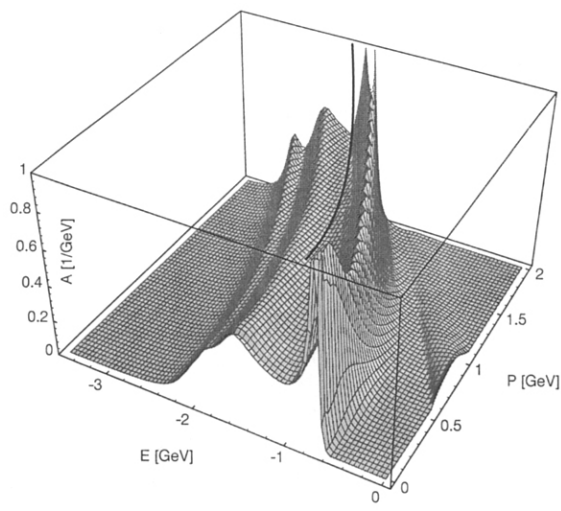


Fig. 2. The  $\gamma^0$  component of the nucleon spectral function for negative energies and pseudoscalar coupling. The cut-off is  $\Lambda = 1.2$  GeV. The line shows the position of the mean-field delta function.

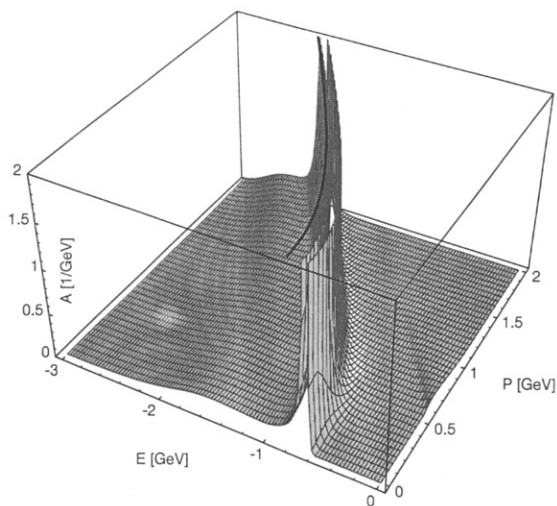


Fig. 3. The  $\gamma^0$  component of the nucleon spectral function for negative energies and pseudovector coupling. The cut-off is  $\Lambda = 1.2$  GeV. The line shows the position of the mean-field delta function.

tion about these spectral functions and distinguish between the two models.

The unexpected results of ref. [8] imply the importance of taking into account the widths of quasi-particle peaks in the medium. To study the broadening of the nucleon peak it is necessary to dress the pion, i.e. take into account the particle-hole excita-

tions. We have not achieved complete self-consistency with both pion and nucleon dressed, but were able to sum a large subclass of diagrams. We start with the (shifted) vacuum solution, which is the approximate high-momentum limit of the self-consistent solution and calculate the in-medium Fock diagram. The pion is dressed using the nucleon propagator obtained in such a way, and then we use the dressed pion to calculate the addition to the nucleon self-energy leading to the broadening of the quasi-particle peak. We also use the form-factor (6), but with  $|k^2|$  replacing  $k^2$ , thus providing suppression also in the Euclidean region. The obtained results, for pseudovector coupling, are shown in fig. 4.

The difference between the two couplings is not so large in this case, the pseudoscalar one giving in general larger widths. The widths of the hole states are quite small and comparable to results obtained in a completely different model [10]. However, the broadening for larger momenta becomes quite significant. For  $p = 0.5$  GeV the width at half maximum is around 80 MeV, and it increases to around 120 MeV for  $p$  around 0.8 GeV, decreasing afterwards to 100 MeV at  $p = 1$  GeV, 70 MeV at 1.5 GeV and 50 MeV at  $p = 2$  GeV. We mention that these widths are hardly sensitive to the values of the cut-off parameters. As one can see from fig. 4 the shift with respect to the

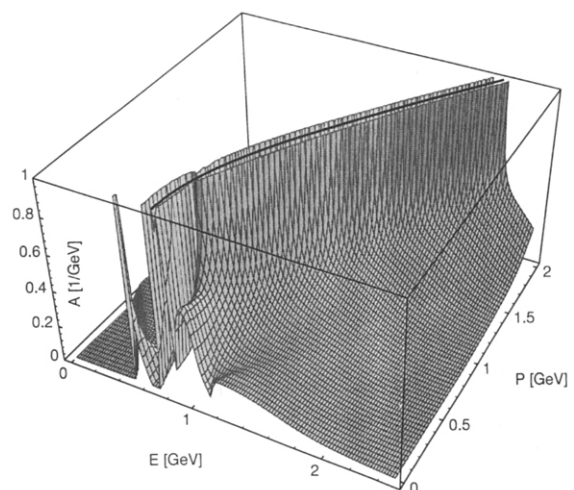


Fig. 4. The  $\gamma^0$  component of the nucleon spectral function for positive energies, with dressed pion and pseudovector coupling. The cut-off parameters are  $\Lambda = 1.2$  GeV and  $\Lambda_\pi = 0.8$  GeV. The line shows the position of the mean-field delta function.

mean-field delta function is very small. We remark that the sum-rule for the  $\gamma^0$  component of the spectral function is satisfied to a few percent in the computation. We also checked the effect of the short-range repulsion (Migdal correction) on our results, but using  $g'_{\text{NN}} = 0.6$  caused only a slight narrowing of the quasi-particle peaks.

### Acknowledgement

This work was supported by the Stichting Fundamenteel Onderzoek der Materie (FOM). We thank Maxim Mostovoy for numerous discussions.

### References

- [1] B.D. Serot and J.D. Walecka, *Adv. Nucl. Phys.* 16 (1986) 1;  
B. ter Haar and R. Malfliet, *Phys. Rep.* 149 (1987) 207.
- [2] R.J. Furnstahl, R.J. Perry and B.D. Serot, *Phys. Rev. C* 40 (1989) 321.
- [3] L. Wilets, in: *Mesons in nuclei*, eds. M. Rho and D. Wilkinson (North-Holland, Amsterdam, 1979) p. 791.
- [4] J.P. Milana and P.J. Siemens, *Phys. Rev. C* 43 (1991) 2377.
- [5] C.L. Korpa and R. Malfliet, to be published.
- [6] P. Henning, *Nucl. Phys. A* 546 (1992) 653.
- [7] P.J. Siemens, M. Soyeur, G.D. White, L.J. Lantto and K.T.R. Davies, *Phys. Rev. C* 40 (1989) 2641.
- [8] L. Xia, P.J. Siemens and M. Soyeur, Coupling and decoupling of pions and deltas in nuclear matter, in: *Proc. Intern. Workshop on Gross properties of nuclei and nuclear excitations XX* (Hirschegg, Austria, January 1992) p. 183.
- [9] A.F. Bielajew and B.D. Serot, *Ann. Phys. (NY)* 156 (1984) 215.
- [10] F. de Jong and R. Malfliet, *Phys. Rev. C* 44 (1991) 998.